Optimal management of immunized portfolios

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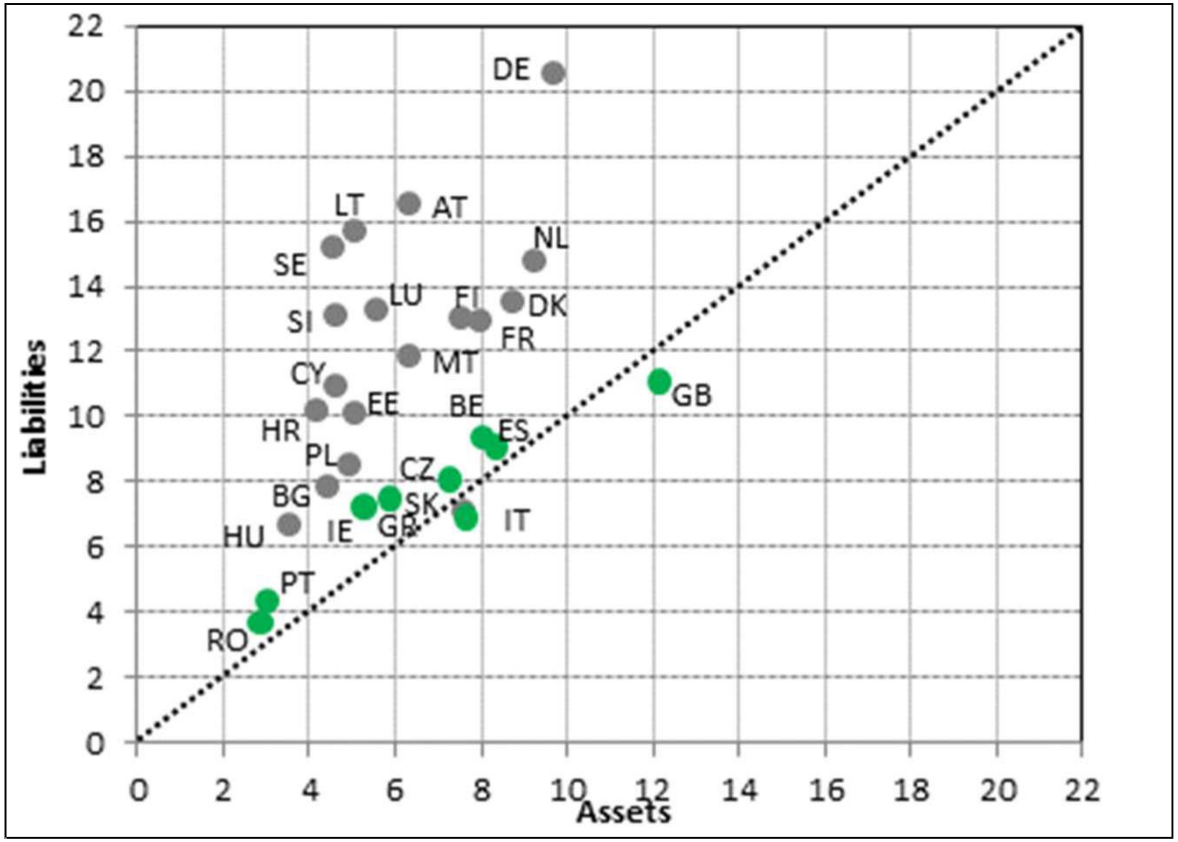
AMASES, 41°
Cagliari, September, 15°, 2017

- Banks: maturity transformation
  - Short term liabilities (deposits) vs long term assets (loans)
- Insurers: long term guarantee provision
  - Long term asset & liabilities ⟹ maturity matching
    cash flow matching (ALM)

- Redington (1952): duration matching (classical «immunization»)
  - Duration as measure of interest rate risk:
    Small, constant, additive changes of the term structure of interest rates
- Fong and Vasicek (1984)
  - For arbitrary changes of the TS also M^2 matters
    M^2 = variance of cash flow times around average (i.e. duration)

$$D = \sum_{j=1}^{m} s_j w_j \qquad w_j = \frac{A_j P(0, s_j)}{\sum_{h=1}^{m} A_h P(0, s_h)} \qquad M^2 = \sum_{j=1}^{m} (s_j - D)^2 w_j$$

Duration of assets and liabilities in the EIOPA Stress test Report (2014)



- Fong and Vasicek (1984) theorem

  A lower bound for the change in value of the portfolio

  If D_A=H (=D):

$$\frac{\Delta A_0(H)}{A_0(H)} \geq -\frac{1}{2} K_0 M_0^2(D)$$

where $A_0(H)$ is the forward value in H (evaluated in 0) of the portfolio, $K_0=\max_\tau (\Delta'_0(\tau))$ and $\Delta'_0(\tau)$=«slope» of the forward rate change

$$\Delta_0(\tau) = r_{FW}(0^+, \tau) - r_{FW}(0, \tau)$$

- Passive strategy: min risk or generalized immunization:
  - Duration matching (H=D) + minimize M_0^2

- Active strategy: optimize risk / return trade-off

$$(1+\overline{R}_0(H))^H \equiv \frac{A_0(H)}{A(0)}$$

current forward annual return

$$\Delta\overline{R}_0(H) \simeq \frac{1}{H}\frac{\Delta A_0(H)}{A_0(H)} \simeq \frac{1}{H} {M_0}^2(H)\Delta S_0(H)$$

where

$$\Delta S_0(H) \equiv \frac{1}{2}\left[{\Delta_0}^2(H) - \Delta_0'(H)\right] \gtreqless 0$$

- Summing up across times

$$R_H(H) - \overline{R}_0(H) = \sum_{t=0}^{H-1} \Delta R_t(H) \simeq \frac{1}{H} \sum_{t=0}^{H-1} M_t^2(H) \Delta S_t(H)$$

But cash flows are bought and sold in «bundles» (coupon bonds)

Therefore:

K bond portfolio

$$\sum_{i=1}^{K} w_i (R_{iH} - \overline{R}_{i0})$$

$$R_{iH} - \overline{R}_{i0} = \frac{1}{H} \sum_{t=0}^{H-1} M_{it}^2(H) \Delta S_t(H) \qquad i = 1, \dots, K$$

$$M_{it}^2(H) \Delta S_t(H) \cong M_{i0}^2(H) \left(\frac{H-t}{H}\right)^3 \Delta S_0(D_{i0}) \left(\frac{H-t}{D_{i0}}\right)$$

$$M_{i0}^2(H) = \sum_{j=1}^{m} (s_j - H)^2 w_{0j} = M_0^2(D_{i0}) + (D_{i0} - H)^2$$

- Summing up across times

$$R_H(H) - \overline{R}_0(H) = \sum_{t=0}^{H-1} \Delta R_t(H) \simeq \frac{1}{H}\sum_{t=0}^{H-1} M_t^2(H)\Delta S_t(H)$$

But cash flows are bought and sold in «bundles» (coupon bonds)

Therefore:

$$\sum_{i=1}^{K} w_i(R_{iH} - \overline{R}_{i0}) \equiv w'R$$

K bond portfolio

w'μ (mean)

w'Σw variance

$$R_{iH} - \overline{R}_{i0} = \frac{1}{H}\sum_{t=0}^{H-1} M_{it}^2(H)\Delta S_t(H) \qquad i = 1, \dots, K$$

$$M_{it}^2(H)\Delta S_t(H) \cong M_{i0}^2(H)\left(\frac{H-t}{H}\right)^3 \Delta S_0(D_{i0})\left(\frac{H-t}{D_{i0}}\right)$$

$$M_{i0}^2(H) = \sum_{j=1}^{m}(s_j - H)^2 w_{0j} = M_0^2(D_{i0}) + (D_{i0} - H)^2$$

- Markowitz-type portfolio selection:

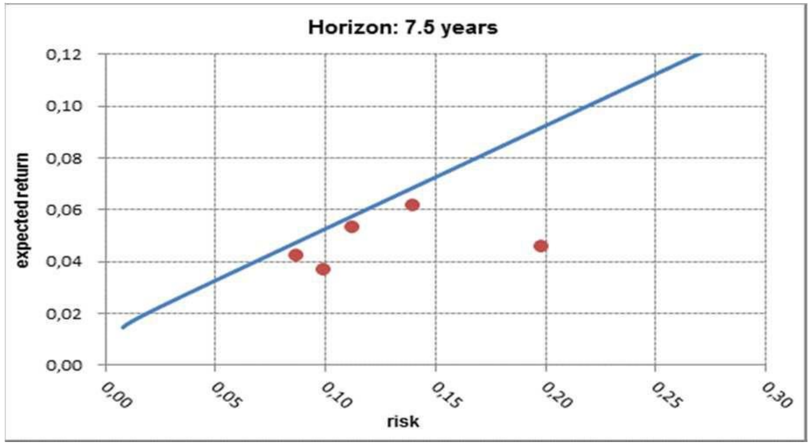
  more return in exchange of more risk

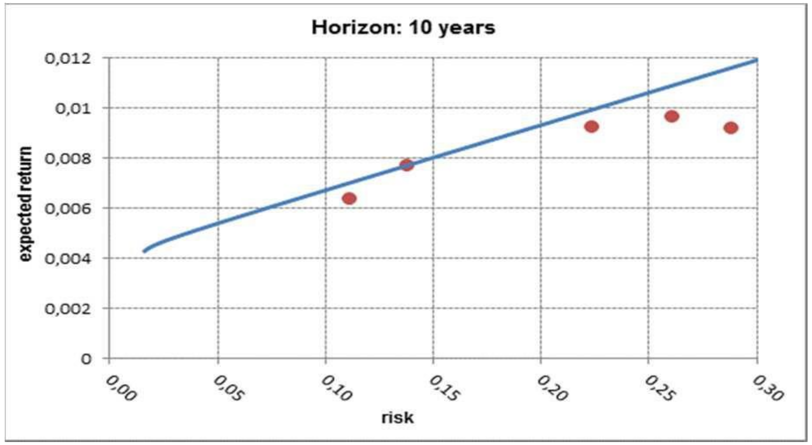
$$\max_{w_i} \boldsymbol{U}(\boldsymbol{w}'\mu, \boldsymbol{w}'\Sigma\boldsymbol{w})$$

$$\begin{cases} \sum_{1\leq i\leq K} w_i \cdot D_{i0} = H \; (target\ duration\ constraint) \\ \sum_{1\leq i\leq K} w_i = 1 \quad (budget\ constraint) \\ w_i \geq 0 \quad (no\ short\ selling) \end{cases}$$

➢ Empirical application

- 10 large life companies (54% of market share) with $D_A \approx D_L$
- Two horizon H=7.5 and H=10 years
- 4 bond benchmarks (3,5,10,20 years to maturity)
- Efficient frontiers: allocation (in)efficiencies





Conclusions and further developments

- Duration matching (classical immunization) is half the job
- Under arbitrary TS movements, cash flow variance (M^2) must be managed
- An immunized portfolio has a risky return which can be optimized in the mean-variance space (efficient frontier)
- A more general application would include multiple liabilities and the whole variety of outstanding (government) bonds
- A higher generalization would include a 3D frontier (risk, return, horizon) in which the insurance product characteristics (horizon etc.) are endogenized.

(full paper in www.ivass.it, Quaderno n.7)